A FAST CELL-SPARSE SOLVER IN FINITE ELEMENT ANALYSES

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Symmetric positive definite equation solvers play a very important role in promoting the efficiency of the finite element analyses (FEA). The focus of this paper is to report the achievement in fast cell sparse solver for symmetric positive definite system of equations. Numerical tests indicate that the cell-sparse storage scheme and the two-level unrolling can improve the performance of symmetric positive definite equation solvers significantly. We consider the following linear system of equations: $\boldsymbol{A}\,\boldsymbol{x} = \boldsymbol{f}$. Here the matrix $\boldsymbol{A}=(A_{ij}) \in R^{neq \times neq}$ is the symmetric positive definite global stiffness matrix arising in FEA. The vectors $\boldsymbol{x} \in R^{neq}$ and $\boldsymbol{f} \in R^{neq}$ stand for the unknown displacement and known load vector, respectively. To solve the above system, one important way is to factorize the matrix $\boldsymbol{A}$ into $\boldsymbol{A}=\boldsymbol{LU}=\boldsymbol{LDL}^{\mathrm{T}}$, where the factors $\boldsymbol{L}=(L_{ij}) \in R^{neq \times neq}$ and $\boldsymbol{D}=\mathrm{diag}(D_{ii}) \in R^{neq \times neq}$ are a lower triangular matrix with unit diagonal and a diagonal matrix, respectively. We proposed the incorporation of the cell-sparse storage scheme and loop-unrolling for the factorization[1,2]. In this paper, the fast cell-sparse solver (FCSS) is re-examined by a few benchmark problems available in literatures.

The test problems are selected from Harwell-Boeing (BCSSTK*)[3] and CYSHELL (S3DK*). The performance of the fast cell-sparse solvers is compared with MA47 available at HSL Archive on the platform: a Dell Pentium III 850 using Compaq Visual Fortran 6.5, with compiler options /architecture:p6p /tune:p6p, 1 GB RAM. All problems were reordered by the METIS_NODEND algorithm for the sparse solvers. Table 1 illustrates the elapsed time for the $\boldsymbol{LDL}^{\mathrm{T}}$ factorization as well as time for solution, i.e., forward reduction and back substitution, on the computational platform. The proposed fast cell-sparse solver needs on the average much less factorization time and less solution time than the MA47.

Table 1: factorization time and solution time on PIII 850

PROBLEM	MA47	FCSS
BCSSTK17	2.03/0.16	1.15/0.11
BCSSTK18	0.71/0.11	0.99/0.11
BCSSTK25	3.13/0.22	3.02/0.11
BCSSTK29	2.47/0.25	2.09/0.11
BCSSTK30	10.99/0.55	4.84/0.27
BCSSTK31	11.37/0.60	8.19/0.27
BCSSTK32	22.19/0.77	8.79/0.33
S3DKQ4M2	99.30/2.09	16.36/0.83
S3DKT3M2	95.99/1.83	15.27/0.72

The numerical tests showed that the fast cell-sparse solver is very efficient in terms of elapsed time and memory requirement and can be used as the default direct solver in FEA.

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References

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